$$y_0,\ y_1,\ y_2,\ \ldots\ldots,\ y_n$$

approaches a limit y, when n increases indefinitely, the sequence

$$\psi(y_0),\ \psi(y_1),\ \psi(y_2),\ \ldots\ldots,\ \psi(y_n)$$

approaches the limit $\psi(y)$, and hence *the identical sequence*

$$k^{y_0},\ k^{y_1},\ k^{y_2},\ \ldots\ldots\ k^{y_n}$$

*approaches a limit.** This limit is *defined* as k^y.† Hence for all real values of y

$$\psi(y)=k^y \text{ and hence } \phi(x)=\log_k(x).$$

EVANSTON, ILLINOIS, *September 21, 1903.*

CONVERSE AND OPPOSITE PROPOSITIONS.

By C. M. HIMEL, Baker-Himel School, Knoxville, Tenn.

Wentworth, in the revised edition of his Plane Geometry, page 5, makes the following confusing statements:

"If a direct proposition and its opposite are true, the converse proposition is true; and if a direct proposition and its converse are true, the opposite proposition is true."

"Thus, if it were true that

1. If an animal is a horse, the animal is a quadruped;
2. If an animal is not a horse, the animal is not a quadruped;

it would follow that

3. If an animal is a quadruped, the animal is a horse.

Moreover, if 1 and 3 are true, then 2 would be true."

The statements should read: *Whether a direct proposition is true or not, if the converse is true the opposite is true; if the opposite is true the converse is true.*

Thus, in the above example, if 1 be true, then 3 would follow; if 3 be true, then 2 would be true. Consider the general case:

1. Proposition: If a is b, then c is d.
2. Converse: If c is d, then a is b.
3. Opposite: If a is not b, then c is not d.

We may prove that if either of the last two, irrespective of the first, is true, the other is also true.

First. Suppose you know that the converse is true, and you wish to prove

*This required proof.

†Cf. Stolz, *loc. cit.*, p. 138; Tannery, *Theorie des fonctions d'une variable*, p. 114.

the opposite. You are given that a is not b, and you wish to prove that c is not d. It is not, because, if it is, then a is b; but a is not b, therefore c is not d.

Second. By a similar proof, if the opposite be true, the converse is true.

For example, from the theorem [compare the proof in Wentworth's Revised Geometry, p. 70, Ex. 55]: If two angles of a triangle are not equal, their bisectors are not equal, one may conclude: If the bisectors of two angles of a triangle are equal, the triangle is isosceles. The proof of the first theorem is, indeed, a proof of the second.

DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

GEOMETRY.

203. Proposed by W. J. GREENSTREET, A. M., Editor of The Mathematical Gazette, Stroud; England.

Show that two parabolae can always be drawn through the vertices of a triangle to touch its circumcircle at a vertex, and that the axes of these pairs of curves are orthogonal. Show that any triangle may be circumscribed by a conic so that the tangents at each vertex is parallel to the opposite side.

No solution has been received.

204. Proposed by ELMER SCHUYLER, B. Sc., Professor of German and Mathematics, Boys' High School, Reading, Pa.

Construct a triangle, having given an angle, the length of its bisector, and the sum of the including sides. (Phillips and Fisher).